JOURNAL OF APPROXIMATION THEORY 82, 468–479 (1995)

Book Reviews

BOOK REVIEW EDITOR: WALTER VAN ASSCHE

Books

V. TOTIK, *Weighted Approximation with Varying Weight*, Lecture Notes in Mathematics **1569**, Springer-Verlag, 1994, vi + 115 pp.

These lecture notes are really a long paper, and a very important one at that. Vilmos Totik has largely solved the problems on approximation by weighted polynomials of the form $w^n P_n$, where P_n is a polynomial of degree at most n, and w is a fixed weight function. The essential feature is that the power of the weight changes as n changes. To many such a problem will seem artificial. However, a solution to this problem has turned out to be essential in the theory of orthogonal polynomials, in incomplete polynomials, and in ordinary weighted approximation.

Many researchers have contributed to this problem, among them P. Borwein, M. von Golitschek, G. G. Lorentz, A. L. Levin, X. Li, the reviewer, G. López, H. N. Mhaskar, E. B. Saff, and R. S. Varga. The most successful earlier method of solving this problem involved replacing w in some sense by an analytic function $\hat{w}$, and then choosing as P_n a Lagrange interpolant to $\hat{w}^{-n}$. The error was estimated by Hermite's contour integral error formula for Lagrange interpolation.

Totik's lecture notes are devoted to a method which proceeds directly from the logarithmic potential function associated with w, thereby avoiding the need to replace w by an analytic weight. So Totik's method applies more generally and is more elegant. It involves carefully discretizing the potential to generate P_n, but the zeros of P_n are shifted slightly into the complex plane to avoid problems on the real line. Other authors tried this earlier, getting partial results, but Totik was really the first to realize that in analyzing the approximation, it is essential to use the fact that the upward normal derivative of the potential at points on the real axis is essentially the Poisson integral of the measure μ_w in the potential. Totik makes considerable use of this clever observation.

A special case of Totik's results is the following: let w be continuous on $\mathbb{R}$ and positive on a set of positive logarithmic capacity, with $\lim_{|x| \to \infty} |x| w(x) = 0$. If the equilibrium measure μ_w associated with w is supported on $[a, b]$ and is positive and continuous on (a, b), then for every continuous function $f: \mathbb{R} \to \mathbb{R}$ that vanishes outside $[a, b]$, one can find polynomials P_n of degree at most n, with

$$\lim_{n \to \infty} \|f - P_n w^n\|_{L_\infty(\mathbb{R})} = 0.$$

This special case of Totik's result includes all earlier results for weights such as $w(x) = x^\theta$ ($x \in [0, 1]$) or $w(x) = \exp(-|x|^\alpha)$ ($x \in \mathbb{R}$).

Totik shows that approximation may not be possible if μ_w vanishes at some point of (a, b). Moreover, he applies his results to weighted approximation by $P_n w_n^n$, where w_n has certain asymptotic properties as $n \to \infty$. As applications of the general theorems, fast decreasing polynomials are discussed, as are asymptotics for orthogonal polynomials for Freud and varying weights, extremal problems, and multipoint Padé approximation.

0021-9045/95 $12.00

These lecture notes are essential reading for anyone interested in orthogonal polynomials, weighted approximation, and potential theory. It is an achievement of lasting value, possibly as significant as the Pollard–Mergelyan–Akhiezer solutions to Bernstein's approximation problem in the 1950s.

DORON LUBINSKY

TOM H. KOORNWINDER (Ed.), *Wavelets: An Elementary Treatment of Theory and Applications*, Series in Approximations and Decompositions 1, World Scientific, Singapore, 1993, xii + 225 pp.

Based on a four-day intensive course given at the CWI, Amsterdam, this collection of papers is a contribution to help a rather broad audience with understanding wavelets. It aims at giving the general ideas of the basic theory, as well as of some important applications, at a level which is adequate for a mixed audience. The need of such a presentation is evident, although (and since) the wavelet literature is tremendously increasing. Everybody who has ever taught a mathematical course in this direction will appreciate having such a guideline, and anyone who applies (discrete) wavelets to analyze and to process data—wherever these may originate from—will welcome this elementary treatment.

The book consists of twelve articles written mainly by scientists from the CWI and the University of Amsterdam. First there are two introductory expositions. *Wavelets: First Steps*, by N. M. Temme, gives an overview of the continuous and the discrete wavelet transform, as well as of multiresolution analysis. It also shows the general principle how to find functions ϕ satisfying a dilation (or scaling) equation and how to construct the corresponding wavelet in case the integer translates of ϕ are orthonormal. *Wavelets: Mathematical Preliminaries*, by P. W. Hemker *et al.*, compiles some mathematical background including Hilbert space notions, Fourier approximation, but also Riesz bases and frames in Hilbert spaces.

Next follow two articles describing the theory of wavelet analysis. *The Continuous Wavelet Transform*, by T. H. Koornwinder, starts with the transformation formula and its corresponding Parseval identity and inversion formula. Localization in the time-frequency domain is discussed, and an exposition is given on how to deal with the continuous wavelet transform from a more abstract point of view referring to unitary representations of locally compact groups. *Discrete Wavelets and Multiresolution Analysis*, by H. J. A. M. Heijmans, gives the construction of a nested sequence of multiresolution spaces by defining the Fourier transform of the basis function in terms of the usual infinite product. Then the corresponding wavelet basis of $L^2(\mathbb{R})$ is constructed, and the recursive formulas for finding the Fourier-wavelet expansion are given. As an example, the Meyer wavelet is shown to satisfy this construction principle.

The following two contributions *Image Compression Using Wavelets*, by P. Nacken, and *Computing with Daubechies' Wavelets*, by A. B. Olde Daalhuis, show the details of the wavelet decomposition and reconstruction formulas in terms of discrete convolution and down-sampling (upsampling, respectively) procedures. The idea of compression based on the wavelet expansion as well as the corresponding reconstruction error is discussed (but not worked out in detail).

Wavelet Bases Adapted to Inhomogeneous Cases, by P. W. Hemker and F. Plantevin, gives some extensions of the wavelet ideas to cases where typical ingredients of wavelet theory are not present. In particular, they discuss the construction of orthonormal (wavelet) bases for L^2-functions on a bounded domain $\Omega \subset \mathbb{R}^n$ satisfying homogeneous Dirichlet boundary conditions, they introduce wavelets associated with the boundary value problem $D^*a\,Du = f$ on $I = [0, 1]$, where $u(0) = u(1) = 0$, with $D = -i(d/dx)$ and a a (bounded and accretive) complex-valued function, and they deal with wavelets on certain irregular meshes.

The remaining five articles are more directed into giving concrete ideas and hints for applications. *Conjugate Quadrature Filters for Multiresolution Analysis and Synthesis*, by